CSCI446/946 Big Data Analytics Week 4 – Lecture: Clustering

School of Computing and Information Technology University of Wollongong Australia Spring 2022

Content

- Brief Recap
 - Hypothesis Testing

- Statistical Methods for Evaluation
 Analysis of Variance (ANOVA)
- Clustering Analysis
 - K-means, DBSCAN, SOM

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- Hypothesis Testing
 - Form an assertion and test it with data
 - Common assumption (there is no difference)
 - Null hypothesis (H_0) vs Alternative hypothesis (H_A)

A hypothesis is formed before validation
 It can define expectations.

- Analysis of the difference of two Means
 - Very common hypothesis test.
 - But simple comparison is often not sufficient.
 - Example: Assume we have two populations, one with mean=-3 and the other with mean=3
 - By comparing the means can we say that the difference between the two **populations** is significant?
 - Answer depends on variance.





• Student's t-test

- Assumptions: Two populations, normally distributed and have a similar variance.
- Welch's t-test
 - Assumptions: Two populations, normally distributed.
- Wilcoxon Rank-Sum Test
 - Assumptions Two populations, not normal distributed.
- ANOVA
 - When: More than two populations.
- Many others: Mann-Whitney test, Kruskal-Wallis test, Fisher's exact test, chi-square test, McNemartest, Friedman test, log rank test, spearman correlation test, Pearson correlation test....

- Wilcoxon Rank-Sum Test Suppose we have the following data:
 Group A: [85, 80, 78, 90, 95]; Group B: [88, 82, 85, 87, 92]
- Step 1: Combine and Rank the Data
 - Combine: [85, 80, 78, 90, 95, 88, 82, 85, 87, 92]
 - Rank: [4.5, 2, 1, 8, 10, 7, 3, 4.5, 6, 9]
- Step 2: Sum the Ranks for Each Group
 - Group A: [4.5, 2, 1, 8, 10]; Sum of ranks *W*₁=4.5+2+1+8+10=25.5
 - Group B: [7, 3, 4.5, 6, 9]; Sum of ranks W_2 =6+3+4.5+5+9=29.5
- Step 3: Choose the Test Statistic
 - W can be either 25.5 or 29.5 depending on the test design, but usually, the smaller sum is used if conducting a one-sided test.
- Step 4: Determine Significance
 - Compare the test statistic W to a critical value from the Wilcoxon ranksum distribution or use a p-value from statistical software.

Brief Recap



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- ANOVA (Analysis of Variance)
 - What if there are more than two populations?
 - Multiple t-test may not perform well now
- A generalization of the hypothesis testing
 - ANOVA tests if any of the population means differ from the other population means
 - Each population is assumed to be normal and have the same variance

• ANOVA (Analysis of Variance)

$$\boldsymbol{H_0}: \boldsymbol{\mu}_1 = \boldsymbol{\mu}_2 = \ldots = \boldsymbol{\mu}_n$$

 $H_A: \mu_i \neq \mu_j$ for at least one pair of i, j

- Compute *F*-test statistic
 - Between-groups mean sum of squares
 - Within-groups mean sum of squares

$$S_B^2 = \frac{1}{k-1} \sum_{i=1}^k n_i \cdot (\overline{x}_i - \overline{x}_0)^2 \qquad S_W^2 = \frac{1}{n-k} \sum_{i=1}^k \sum_{j=1}^{n_i} (x_{ij} - \overline{x}_j)^2$$

• ANOVA (Analysis of Variance)



- ANOVA (Analysis of Variance)
 - Measures how different the means are relative to the variability within each group
 - The larger the *F*-test statistic, the greater the likelihood that the difference of means are due to something other than chance alone
 - The *F*-test statistic follows an <u>*F*-distribution</u>

$$F = \frac{S_B^2}{S_W^2}$$

• ANOVA (Analysis of Variance)

• Shall we accept or reject the null hypothesis?

- ANOVA (Analysis of Variance)
 - One-Way ANOVA
 - Compares means across different groups based on a single independent variable (factor).
 - e.g. Comparing the mean test scores of students across different teaching methods (Method A, Method B, Method C).
 - Two-Way ANOVA:
 - Compares means across groups based on two independent variables (factors), and can also evaluate the interaction between the factors.
 - e.g. Comparing test scores based on teaching methods (Factor 1) and study times (Factor 2).

- Limitations of ANOVA (Analysis of Variance)
 - Assumptions
 - Normality: Data should be approximately normally distributed.
 - Homogeneity of Variances: Variances within each group should be equal (tested using Levene's test).
 - Independence: Observations should be independent of each other.
 - Limitations:
 - Sensitivity to Outliers: Outliers can affect the F-statistic and lead to misleading results.
 - Assumes Equal Variances: Violations of this assumption can impact the validity of the results.
 - Identifies Differences but Not Specifics: ANOVA indicates whether a difference exists but does not specify which groups are different without further tests (post-hoc).

- ANOVA (Analysis of Variance)
 - Additional tests for each pair of groups
 - Tukey's Honest Significant Difference (HSD)

```
TukeyHSD(model)
Tukey multiple comparisons of means
95% family-wise confidence level
```

```
Fit: aov(formula = purchase_amt ~ offers, data = offertest)
```

```
$offers
```

diff lwr upr p adj offer1-nopromo 40.961437 33.4638483 48.45903 0.0000000 offer2-nopromo 48.120286 40.5189446 55.72163 0.0000000 offer2-offer1 7.158849 -0.4315769 14.74928 0.0692895

• Tukey's Honest Significant Difference (HSD)

Assumptions

 Norm + equal variance + sample sizes are approximately equal (though it can still be used if they are not).

Perform ANOVA test

 establish whether there is a significant difference between the means of the groups

Calculation of the HSD

 Critical value from studentized range distribution, Mean square within groups(from ANOVA), number of groups

Decision Rule

- For each pair of means, calculate the absolute difference.
- Compare the absolute difference to the HSD value.
- If the absolute difference is greater than the HSD, the pair of means is considered significantly different.

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Clustering

- Overview of Clustering
- K-means clustering
 - Overview of the Method
 - Determining the Number of Clusters
 - Diagnostics
 - Reasons to Choose and Cautions
- Additional Algorithms
 - Density Based Clustering
 - Self-Organize Map (SOM)
 - Hierarchical Clustering

Overview of Clustering

- Supervised vs. Unsupervised Techniques
 Labelled data vs. Unlabelled data
- Unsupervised Techniques
 - Refers to the problem of finding hidden structure within unlabelled data
 - Clustering, density estimation, dimensionality reduction, etc.
- Clustering is an unsupervised technique

K-means Clustering

- Given a collection of m objects each with n measurable attributes
 - Mathematically, $\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_m \in \mathbb{R}^n$
 - Each object is a point in an n-dimensional space



K-means Clustering

 For a chosen value of k, identify k clusters of objects based on the objects' proximity to the centre of the k groups



K-means Clustering

- Use Cases
 - Often used as a lead-in to classification
 - Once clusters are identified, labels can be applied to each cluster to do classification
 - Find out whether the data is organized in cohorts, and how the cohorts align with labels (if available)
- Applications
 - Image Processing
 - Medical (Clustering patients)
 - Customer grouping (find similar customers)

Overview of K-means Clustering

- Three steps
 - Choose a value of k, create k centroids, then initialize them by "guessing" their value.
 - Use k-random selected data points to initialize the centroids. (pick a suitable random algorithm)
 - 2. Compute the distance from each data point to each centroid. Assign each point to the closest centroid.
 - 3. Update the centroid of each cluster
- Repeat Steps 2 and 3 until convergence, i.e., centroids don't change.

Overview of K-means Clustering



Overview of K-means Clustering

• Compute the Euclidean distance

$$d(\mathbf{x}, \mathbf{y}) = \|\mathbf{x} - \mathbf{y}\|_2 = \sqrt{\sum_{i=1}^{n} (x_i - y_i)^2}$$

Compute the centroid for a cluster (centre of gravity)

$$\bar{\mathbf{x}} = \frac{\sum_{i=1}^{m} \mathbf{x}_i}{m}$$

Determine the Number of Clusters

- What value of k shall be selected?
 - A reasonable guess, some predefined requirement
 - k-1, k, or k+1?
- Within Sum of Squares (WSS)
 - A heuristic
 - Sum of the squares of the distances between each data point and the closest centroid

$$J = \sum_{i=1}^{n} \sum_{j=1}^{k} r_{ij} \|\mathbf{x}_i - \bar{\mathbf{x}}_j\|_2^2; \quad r_{ij} \in \{0, 1\}$$

Determine the Number of Clusters

• An optimization point of view

- A combinatorial partition problem

$$J = \sum_{i=1}^{n} \sum_{j=1}^{k} r_{ij} \|\mathbf{x}_i - \bar{\mathbf{x}}_j\|_2^2; \quad r_{ij} \in \{0, 1\}$$

$$\{r_{ij}^*\} = \arg\min_{r_{ij} \in \{0,1\}} J$$

Determine the Number of Clusters

• Within Sum of Squares (WSS)

$$J = \sum_{i=1}^{n} \sum_{j=1}^{k} r_{ij} \|\mathbf{x}_i - \bar{\mathbf{x}}_j\|_2^2; \quad r_{ij} \in \{0, 1\}$$



Number of Clusters

Using R to Perform K-mean Clustering

• Task is to

Group 620 high school seniors based on their grades in "English", "Math", and "Science"

```
grade_input = as.data.frame(read.csv('grades_km_input.csv'))
kmdata_orig =
as.matrix(grade_input[,c("Student","English","Math","Science")])
kmdata <- kmdata_orig[,2:4]</pre>
```

Clust	ering-demo.R	. × 🔛 k	mdata ×		kmdata_orig ×	grade_input ×							
⟨⊐□⟩ 2 Filter													
-	English 🗦	Math $\hat{~}$	Science	÷									
1	99	96		97									
2	99	96		97									
3	98	97		97									
4	95	100		95									
5	95	96		96									
6	96	97		96									
7	100	96		97									

Using R to Perform K-mean Clustering

Compute and plot WSS to choose k value

wss <- numeric(15)
for (k in 1:15) wss[k] <- sum(kmeans(kmdata, centers=k, nstart=25)\$withinss)</pre>

plot(1:15, wss, type = "b", xlab="Number of Clusters", ylab="Within Sum of Squares")



nstart option attempts multiple initial configurations and reports on the best one. For example, adding nstart=25 will generate 25 initial random centroids and choose the best one for the algorithm

Using R to Perform K-means Clustering

Perform K-means Clustering

```
km = kmeans(kmdata,3, nstart=25)
km
```

```
K-means clustering with 3 clusters of sizes 158, 218, 244
```

Using R to Perform K-means Clustering

• Perform K-means Clustering

[521] 2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
	2	2	2	2	2	2	2	2	2	2																				
[561] 2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
	2	2	2	2	2	2	2	2	2	2																				
[601] 3	3	2	2	3	3	3	3	1	1	3	3	3	2	2	3	2	3	3	3										
<pre>Within cluster sum of squares by cluster: [1] 6692.589 34806.339 22984.131 (between_SS / total_SS = 76.5 %) c(wss[3], sum(km\$withinss)) Available components: [1] 64483.06 64483.06</pre>																														
				-											L															
[1] "cluster" "centers" "totss"									"withinss" "tot.withinss"																					
[6] "betweenss" "size" "iter"										"ifault"																				

Using R to Perform K-means Clustering

• Visualize the identified clusters and centroids



Diagnostics – K-means (clustering)

- The following questions shall be asked
 - Are the clusters well separated from each other?
 - Do any of the clusters have only a few points?
 - Do any of the centroids appear to be too close to each other?



Diagnostics

- A principle
 - If using more clusters does not better distinguish the groups, it is almost certainly better to go with fewer clusters



- Several decisions that must be made
 - What object attributions shall be included in clustering analysis?
 - What unit of measure shall be used for each attribute?
 - Do the attributes need to be rescaled?
 - One attribute could have a disproportionate effect

May lead back to Phase 2 data preparation!

- Object attributes
 - Whether it will be known for a new object?
 - Best to reduce the number of attributes to the extent of possible
 - Avoid using too many variables (Why?)
 - Avoid using several similar variables (Why?)
- Identify any highly correlated attributes
- Feature selection, PCA, etc.

Identify any highly correlated attributes



R-Squared or Pearson's r

What is your observation?

Fig. Scatterplot matrix for 7 attributes

• Units of measure could affect clustering result



- Rescaling attributes affect clustering result
 - Divide each attribute by its standard deviation
 - Normalisation: mean=0, sdev=1, particularly when Euclidean distance is used



Additional Algorithms

- K-means clustering is easily applied to numeric data where the concept of distance can naturally be applied
- K-modes handles categorical data
 - Use the number of differences in the respective components of the attributes
 - What is the distance between (a,b,e,d) and (d,d,d,d)?
 - Implemented by the kmode() function
- Caution: Sometimes it is better to convert categorial (or symbolic) data to numerical i.e. {hot, warm, cold} to {1,0,-1}, or use one-hot encoding.
 - Understand why!
 - Understand how to encode categorial values.

Additional Considerations

- Despite its popularity, K-means has problems:
 - When data contains noise and/or outliers
 - When clusters have non-globular shapes
 - Cluster shape affect the choice of the distance metric
 - When clusters vary in densities
 - When clusters differ significantly in size
 - Can reveal "empty" clusters
 - Sensitive to the starting positions of the initial centroids
 - Sunning multiple times with different initialization and choose the one with lowest WSS
- Know your data (i.e via visualization) to verify whether Kmeans is suitable.

Density Based Clustering

- Density-based clustering locates regions of high density that are separated from one another by regions of low density.
- In other words, clusters are dense regions in the data space, separated by regions of lower object density
- Major features of density-based clustering:
 - Discover clusters of arbitrary shape
 - Handle and identify noise
 - Need density parameters as termination condition

 Density is estimated for a particular point in the data set by counting the number of points within a specified radius, *Eps*, of that point. This includes the point itself.



- Example: the number of points within a radius of *Eps* of point A is 7, including A itself.
 - The density of A is 7.

- Given a density threshold (*MinPts*) and a radius (*Eps*), the points in a dataset are classified into three types: core point, border point, and noise point.
 - Core points: Point whose density >= MinPts
 - Core points are in the interior of a density-based cluster.



Example: If *MinPts* = 6 then A is a core point because its density = 7 (7>6)

- Three types: core point, border point, and noise point.
 - A border point is not a core point but falls within the neighborhood of a core point.



Example:

- The density of B is 4 and less than *MinPts* = 6, so B is not a core point.
- But B falls within the neighbor of A (a core point).
- So, B is a border point.

- Three types: core point, border point, and noise point.
 - A noise point is any point that is neither a core point nor a border point.



Example:

- The density of C is 3 which is less than *MinPts* = 6, so C is not a core point.
- C doesn't fall within the neighborhood of any core point, so it is not a border point.
- So, C is a noise point..

- Steps of DBSCAN to identify clusters
 - Step 1: Label each point as either core, border, or noise point.
 - Step 2: Mark each group of Eps connected core points as a separate cluster.
 - Step 3: Assign each border point to one of the clusters of its associate core points.

DBScan Example

Original Points



Mark core, border and noise points

Mark connected core points

DBScan Properties

- DBSCAN:
 - Resistant to noise and outliers
 - Can handle clusters of different shapes and sizes
 - Computational complexity is similar to K-means
- When DBSCAN does not work well
 - Varying densities
 - Can be overcome by using sampling
 - Sparse and high-dimensional data
 - Can be overcome by using topology preserving dimension reduction techniques.

- Self organizing maps are a type of Neural Network (NN).
- Unsupervised algorithm.
- Project high dimensional data onto a ndimensional display space (the feature map).
 – Commonly n=2
- Topology preserving mappings & clustering.
 - Data that is "similar" within the input space remain "close" to each other in the display space

- Self-organizing maps have two layers:
 - An input layer and
 - An output layer called the feature map.
- The feature map consists of neurons.
 - organized on a regular grid.
 - Unlike other ANN types, the neuro in a SOM don't have an activation function.
- Each neuron in a SOM is assigned a weight vector with the same dimensionality as the input space.



- The weights in a SOM are trained in a twostep algorithm:
 - Step 1: Competitive step
 - Every neuron is examined to calculate which one's weights is most similar to the input vector. The winning neuron is known as the Best Matching Unit (BMU).
 - Step 2: Cooperative step
 - The weights of the BMU and the weights of the neighboring neurons is updated.

- Training Algorithm:
 - 1. Each neuron's weights is initialized with random values.
 - 2. A sample is chosen at random from the set of training data.
 - 3. Find the BMU.
 - 4. Identify the neighbourhood of the BMU. The size of the neighborhood decreases over time.
 - 5. Update the weights of the BMU and all of its neighbors so that they become more similar to the sample vector. The closer a node is to the BMU, the more its weights get altered and the farther away the neighbor is from the BMU, the less it is updated.
- Step 2 through to 5 are repeated N times.
 - Normally N is a multiple of the number of training samples.

#Step 1: Prepare the data import pandas as pd import numpyas np from sklearn.utilsimport shuffle

#Step 2: Preprocessthe data
from sklearn.model_selectionimport train_test_split

myseed=7 #Seed for the random number generator

```
#Remove irrelevant features, and targets from df
dropList= ['Patient ID', 'Survival months']
for item in dropList:
df.drop(item, axis=1, inplace=True)
```

```
#Scale the data?
#from sklearnimport preprocessing
#scaling = preprocessing.MinMaxScaler()
#data = scaling.fit transform(data)
```

```
#Create a train, test, and validation set
X, X_tst, Y, Y_tst= train_test_split(df, target, test_size=.333,
random_state=myseed)
X_trn, X_val, Y_trn, Y_val= train_test_split(X, Y, test_size=.5,
random_state=myseed)
```

```
X_trn= X_trn.to_numpy()
X_tst= X_tst.to_numpy()
X val= X val.to numpy()
```

#Step 3: Train the SOM
from myminisom import MiniSom #see Moodle site for myminisom

```
#Create the SOM
som_shape = (100, 100) #define the size of the som
som = MiniSom(som_shape[0], som_shape[1], X_trn.shape[1],
sigma=som_shape[0]/2, learning_rate=.9,
neighborhood_function='gaussian', random_seed=myseed)
#initialize the SOM, then train it
epochs=40
som.pca_weights_init(X_trn)
som.train random(X trn, epochs * len(X trn), verbose=True)
```

```
#Find the BMU for each sample
BMU_trn = np.array([som.winner(x) for x in X_trn])
BMU_class0 = BMU_trn[Y_trn==0]
BMU_class1 = BMU_trn[Y_trn==1]
```

#Step 4: Plot some results (density map of all samples)
import matplotlib.pyplot as plt
from copy import copy

```
densitymap = np.zeros(som_shape)
for row in range(0,BMU_trn.shape[0]):
    x,y = BMU_trn[row]
    densitymap[y,x] += 1
```

```
densitymap[densitymap==0]=np.nan #mark zero values with nan
my_cmap = copy(plt.cm.jet)
my_cmap.set_bad(color=(1,1,1)) #plot nan in white color
plt.imshow(densitymap, cmap=my_cmap, interpolation="none",
origin="lower", aspect=0.75)
plt.colorbar()
plt.title('Mapping density')
plt.show()
```

```
#density map of all samples from class 1
import matplotlib.pyplot as plt
densitymap = np.zeros(som_shape)
for row in range(0,BMU_class1.shape[0]):
    x,y = BMU_class1[row]
    densitymap[y,x] += 1
```

densitymap[densitymap==0]=np.nan #mask zero
values

plt.imshow(densitymap, cmap=my_cmap, interpolation="none", origin="lower", aspect=0.75)

```
plt.colorbar()
plt.title('Mapping density (class 1)')
plt.show()
```

Self-Organizing Maps – An Example



Self-Organizing Maps – An Example



Self-Organizing Maps – An Example



Self-Organizing Maps - Note

Each neuron clusters samples that are mapped to it.

-n x m clusters (size of the SOM)

- A group of neurons form larger cluster
 - Cluster analysis needed to detect these.

#Compute the quantization error

qerr = som.quantization_error(X_trn)

qerr

7.454546962215053

#inspect some weights

som.get_weights()[1,1]

array([1.45920861e+00, 2.94414892e+00, 1.58042285e+00, 1.98123243e+00, 5.60639639e+01, 1.49714974e+02, 2.37928820e-01, 2.06261914e+02, 5.05750682e+02, 1.57189567e+00, 1.03022154e+00, 8.98461177e+00, 1.20844111e+00, 1.33963081e+00, 5.28842982e+01, 4.74582306e+01])

- SOMs are an excellent choice for data visualization
- Many visualization techniques
 - From exploratory data analytics
 - Dimension reduction techniques
 - i.e. PCA, t-SNE, SOM,...
- Why use Self-Organizing Maps (SOMs) in BDA?
 - Topology preservation (unlike PCA)
 - Able to deal with new data & missing values (unlike t-SNE)
 - Can reduces the amount of information that needs to be evaluated
 - Produces prototypes that represent the full set of attributes with their original meaning (unlike PCA)

— ...

- When not to use SOMs in BDA:
 - -When the data is very sparse
 - -When cardinality (limited resolution) of the map is a problem.
 - -When multi-core compute infrastructure is unavailable.

Hierarchical Clustering

- Hierarchical Clustering (hclust())
 - Hierarchical agglomerative clustering
 - Hierarchical divisive clustering



- 1. Each object is initially treated as a cluster
- 2. The clusters are then combined with the most similar cluster in each step
- 3. This process is repeated until one cluster (containing all objects) exists

Computationally very expensive $O(n^2)$ to $O(n^3)$ and thus rarely used in Big Data analytics.



Images Courtesy of Google Image