# **CSCI446/946 Big Data Analytics** Week 4 – Lecture: Clustering

School of Computing and Information Technology University of Wollongong Australia Spring 2022

### Content

- Brief Recap
	- Hypothesis Testing

- Statistical Methods for Evaluation – Analysis of Variance (ANOVA)
- Clustering Analysis
	- K-means, DBSCAN, SOM

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- Hypothesis Testing
	- Form an assertion and test it with data
	- Common assumption (there is no difference)
	- Null hypothesis (*H0*) vs Alternative hypothesis (*HA*)

• A hypothesis is formed before validation – It can define expectations.

- Analysis of the difference of two Means
	- Very common hypothesis test.
	- But simple comparison is often not sufficient.
	- Example: Assume we have two populations, one with mean=-3 and the other with mean=3
		- By comparing the means can we say that the difference between the two **populations** is significant?
		- Answer depends on variance.





#### • Student's t-test

- Assumptions: Two populations, normally distributed and have a similar variance.
- Welch's t-test
	- Assumptions: Two populations, normally distributed.
- Wilcoxon Rank-Sum Test
	- Assumptions Two populations, not normal distributed.
- ANOVA
	- When: More than two populations.
- Many others: Mann-Whitney test, Kruskal-Wallis test, Fisher's exact test, chi-square test, McNemartest, Friedman test, log rank test, spearman correlation test, Pearson correlation test….

- Wilcoxon Rank-Sum Test Suppose we have the following data:
	- Group A: [85, 80, 78, 90, 95]; Group B: [88, 82, 85, 87, 92]
- Step 1: Combine and Rank the Data
	- Combine: [85, 80, 78, 90, 95, 88, 82, 85, 87, 92]
	- Rank: [4.5, 2, 1, 8, 10, 7, 3, 4.5, 6, 9]
- Step 2: Sum the Ranks for Each Group
	- Group A:  $[4.5, 2, 1, 8, 10]$ ; Sum of ranks  $W_1$ =4.5+2+1+8+10=25.5
	- Group B:  $[7, 3, 4.5, 6, 9]$ ; Sum of ranks  $W_2$ =6+3+4.5+5+9=29.5
- Step 3: Choose the Test Statistic
	- W can be either 25.5 or 29.5 depending on the test design, but usually, the smaller sum is used if conducting a one-sided test.
- Step 4: Determine Significance
	- $-$  Compare the test statistic W to a critical value from the Wilcoxon ranksum distribution or use a p-value from statistical software.

#### Brief Recap



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- ANOVA (Analysis of Variance)
	- What if there are more than two populations?
	- Multiple *t*-test may not perform well now
- A generalization of the hypothesis testing
	- ANOVA tests if any of the population means differ from the other population means
	- Each population is assumed to be normal and have the same variance

• ANOVA (Analysis of Variance)

$$
\mathbf{H}_{o}:\mu_{1}=\mu_{2}=\ldots=\mu_{n}
$$

 $H_{\mathbf{A}}$ :  $\mu_i \neq \mu_j$  for at least one pair of *i*, *j* 

- Compute *F-*test statistic
	- Between-groups mean sum of squares
	- Within-groups mean sum of squares

$$
S_B^2 = \frac{1}{k-1} \sum_{i=1}^k n_i \cdot (\overline{x}_i - \overline{x}_0)^2 \qquad S_W^2 = \frac{1}{n-k} \sum_{i=1}^k \sum_{j=1}^{n_i} (x_{ij} - \overline{x}_i)^2
$$

• ANOVA (Analysis of Variance)



- ANOVA (Analysis of Variance)
	- Measures how different the means are relative to the variability within each group
	- The larger the *F*-test statistic, the greater the likelihood that the difference of means are due to something other than chance alone
	- The *F*-test statistic follows an *F*[-distribution](https://en.wikipedia.org/wiki/F-distribution)

$$
\mathsf{F}=\frac{\mathsf{S}^2_{\mathsf{B}}}{\mathsf{S}^2_{\mathsf{W}}}
$$

• ANOVA (Analysis of Variance)

```
# fit ANOVA test
model \lt- aov(purchase amt \sim offers, data=offertest)
summary (model)Df Sum Sq Mean Sq F value Pr(>F)
           2 225222 112611 130.6 <2e-16 ***
offers
Residuals 497 428470 862
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```
• Shall we accept or reject the null hypothesis?

- ANOVA (Analysis of Variance)
	- One-Way ANOVA
		- Compares means across different groups based on a single independent variable (factor).
		- e.g. Comparing the mean test scores of students across different teaching methods (Method A, Method B, Method C).
	- Two-Way ANOVA:
		- Compares means across groups based on two independent variables (factors), and can also evaluate the interaction between the factors.
		- e.g. Comparing test scores based on teaching methods (Factor 1) and study times (Factor 2).

- Limitations of ANOVA (Analysis of Variance)
	- Assumptions
		- Normality: Data should be approximately normally distributed.
		- Homogeneity of Variances: Variances within each group should be equal (tested using Levene's test).
		- Independence: Observations should be independent of each other.
	- Limitations:
		- Sensitivity to Outliers: Outliers can affect the F-statistic and lead to misleading results.
		- Assumes Equal Variances: Violations of this assumption can impact the validity of the results.
		- Identifies Differences but Not Specifics: ANOVA indicates whether a difference exists but does not specify which groups are different without further tests (post-hoc).

- ANOVA (Analysis of Variance)
	- Additional tests for each pair of groups
	- Tukey's Honest Significant Difference (HSD)

```
TukeyHSD (model)
  Tukey multiple comparisons of means
    95% family-wise confidence level
```

```
Fit: aov(formula = purchase amt ~ offers, data = offertest)
```

```
$offers
```
diff lwr upr padj offer1-nopromo 40.961437 33.4638483 48.45903 0.0000000 offer2-nopromo 48.120286 40.5189446 55.72163 0.0000000 offer2-offer1 7.158849 -0.4315769 14.74928 0.0692895

#### • Tukey's Honest Significant Difference (HSD)

#### – Assumptions

- Norm + equal variance + sample sizes are approximately equal (though it can still be used if they are not).
- Perform ANOVA test
	- establish whether there is a significant difference between the means of the groups

#### – Calculation of the HSD

- Critical value from studentized range distribution, Mean square within groups(from ANOVA), number of groups
- Decision Rule
	- For each pair of means, calculate the absolute difference.
	- Compare the absolute difference to the HSD value.
	- If the absolute difference is greater than the HSD, the pair of means is considered significantly different.

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# **Clustering**

- Overview of Clustering
- K-means clustering
	- Overview of the Method
	- Determining the Number of Clusters
	- Diagnostics
	- Reasons to Choose and Cautions
- Additional Algorithms
	- Density Based Clustering
	- Self-Organize Map (SOM)
	- Hierarchical Clustering

# Overview of Clustering

- Supervised vs. Unsupervised Techniques – Labelled data vs. Unlabelled data
- Unsupervised Techniques
	- Refers to the problem of finding hidden structure within unlabelled data
	- Clustering, density estimation, dimensionality reduction, etc.
- Clustering is an unsupervised technique

# K-means Clustering

- Given a collection of m objects each with n measurable attributes
	- Mathematically,  $\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_m \in \mathbb{R}^n$
	- Each object is a point in an n-dimensional space



# K-means Clustering

• For a chosen value of k, identify k clusters of objects based on the objects' proximity to the centre of the k groups



# K-means Clustering

- Use Cases
	- Often used as a lead-in to classification
		- Once clusters are identified, labels can be applied to each cluster to do classification
	- Find out whether the data is organized in cohorts, and how the cohorts align with labels (if available)
- Applications
	- Image Processing
	- Medical (Clustering patients)
	- Customer grouping (find similar customers)

# Overview of K-means Clustering

- Three steps
	- 1. Choose a value of k, create k centroids, then initialize them by "guessing" their value.
		- Use k-random selected data points to initialize the centroids. *(pick a suitable random algorithm)*
	- 2. Compute the distance from each data point to each centroid. Assign each point to the closest centroid.
	- 3. Update the centroid of each cluster
- Repeat Steps 2 and 3 until convergence, i.e., centroids don't change.

## Overview of K-means Clustering



### Overview of K-means Clustering

• Compute the Euclidean distance

$$
d(\mathbf{x}, \mathbf{y}) = \|\mathbf{x} - \mathbf{y}\|_2 = \sqrt{\sum_{i=1}^n (x_i - y_i)^2}
$$

• Compute the centroid for a cluster (centre of gravity)

$$
\bar{\mathbf{x}} = \frac{\sum_{i=1}^{m} \mathbf{x}_i}{m}
$$

## Determine the Number of Clusters

- What value of k shall be selected?
	- A reasonable guess, some predefined requirement
	- k-1, k, or k+1?
- Within Sum of Squares (WSS)
	- A heuristic
	- Sum of the squares of the distances between each data point and the closest centroid

$$
J = \sum_{i=1}^{n} \sum_{j=1}^{k} r_{ij} ||\mathbf{x}_i - \bar{\mathbf{x}}_j||_2^2; \quad r_{ij} \in \{0, 1\}
$$

### Determine the Number of Clusters

• An optimization point of view

– A combinatorial partition problem

$$
J = \sum_{i=1}^{n} \sum_{j=1}^{k} r_{ij} ||\mathbf{x}_i - \bar{\mathbf{x}}_j||_2^2; \quad r_{ij} \in \{0, 1\}
$$

$$
\{r_{ij}^*\} = \arg\min_{r_{ij}\in\{0,1\}} J
$$

#### Determine the Number of Clusters

• Within Sum of Squares (WSS)

$$
J = \sum_{i=1}^{n} \sum_{j=1}^{k} r_{ij} ||\mathbf{x}_i - \bar{\mathbf{x}}_j||_2^2; \quad r_{ij} \in \{0, 1\}
$$



Number of Clusters

#### Using R to Perform K-mean Clustering

• Task is to

#### – Group 620 high school seniors based on their grades in "English", "Math", and "Science"

```
grade input = as.data.frame(read.csv('grades km input.csv'))
kmdata \sigmarig =
as.matrix(grade_input[,c("Student","English","Math","Science")])
kmdata <- kmdata orig[,2:4]
```


#### Using R to Perform K-mean Clustering

#### • Compute and plot WSS to choose k value

 $wss < -$  numeric(15) for (k in 1:15) wss[k]  $\le$  sum(kmeans(kmdata, centers=k, nstart=25)\$withinss)

plot(1:15, wss, type = "b", xlab="Number of Clusters", ylab="Within Sum of Squares")



**nstart** option attempts multiple initial configurations and reports on the best one. For example, adding nstart=25 will generate 25 initial random centroids and choose the best one for the algorithm

#### Using R to Perform K-means Clustering

#### • Perform K-means Clustering

```
km = kmeans(kmdata, 3, nstart=25)km
```

```
K-means clustering with 3 clusters of sizes 158, 218, 244
```

```
Cluster means:
  English Math Science
1 97.21519 93.37342 94.86076
2 73.22018 64.62844 65.84862
3 85.84426 79.68033 81.50820
```

```
Clustering vector:
 [41] 1 1
                1111111111111111111111111
           \mathbf{1}\mathbf{1}\mathbf{1}1111111111
 [81] 1 11 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
             \mathbf{1}\mathbf{1}\mathbf{1}1
                    \mathbf{1}11111111
      \overline{1}[121] 1
                 11111111111111111111111
                \mathbf{1}3 3 3 3 3 3 3 3
     3
      3
```
#### Using R to Perform K-means Clustering

#### • Perform K-means Clustering



#### Using R to Perform K-means Clustering

• Visualize the identified clusters and centroids



# Diagnostics – K-means (clustering)

- The following questions shall be asked
	- Are the clusters well separated from each other?
	- Do any of the clusters have only a few points?
	- Do any of the centroids appear to be too close to each other?



# **Diagnostics**

- A principle
	- If using more clusters does not better distinguish the groups, it is almost certainly better to go with fewer clusters



- Several decisions that must be made
	- What object attributions shall be included in clustering analysis?
	- What unit of measure shall be used for each attribute?
	- Do the attributes need to be rescaled?
		- One attribute could have a disproportionate effect

May lead back to Phase 2 data preparation!

- Object attributes
	- Whether it will be known for a new object?
	- Best to reduce the number of attributes to the extent of possible
		- Avoid using too many variables (Why?)
		- Avoid using several similar variables (Why?)
- Identify any highly correlated attributes
- Feature selection, PCA, etc.

• Identify any highly correlated attributes



**R-Squared** or **Pearson's r**

#### What is your observation?

Fig. Scatterplot matrix for 7 attributes

• Units of measure could affect clustering result



- Rescaling attributes affect clustering result
	- Divide each attribute by its standard deviation
	- Normalisation: mean=0, sdev=1, particularly when Euclidean distance is used



# Additional Algorithms

- K-means clustering is easily applied to numeric data where the concept of distance can naturally be applied
- K-modes handles categorical data
	- Use the number of differences in the respective components of the attributes
		- What is the distance between (a,b,e,d) and (d,d,d,d)?
	- Implemented by the kmode() function
- Caution: Sometimes it is better to convert categorial (or symbolic) data to numerical i.e. {hot, warm, cold} to {1,0,- 1}, or use one-hot encoding.
	- Understand why!
	- Understand how to encode categorial values.

# Additional Considerations

- Despite its popularity, K-means has problems:
	- When data contains noise and/or outliers
	- When clusters have non-globular shapes
		- Cluster shape affect the choice of the distance metric
	- When clusters vary in densities
	- When clusters differ significantly in size
	- Can reveal "empty" clusters
	- Sensitive to the starting positions of the initial centroids
		- Sunning multiple times with different initialization and choose the one with lowest WSS
- Know your data (i.e via visualization) to verify whether Kmeans is suitable.

# Density Based Clustering

- Density-based clustering locates regions of high density that are separated from one another by regions of low density.
- In other words, clusters are dense regions in the data space, separated by regions of lower object density
- Major features of density-based clustering:
	- Discover clusters of arbitrary shape
	- Handle and identify noise
	- Need density parameters as termination condition

• Density is estimated for a particular point in the data set by counting the number of points within a specified radius,  $Eps$ , of that point. This includes the point itself.



- Example: the number of points within a radius of  $Eps$  of point A is 7, including A itself.
	- The density of A is 7.

- Given a density threshold ( $MinPts$ ) and a radius ( $Eps$ ), the points in a dataset are classified into three types: core point, border point, and noise point.
	- $-$  Core points: Point whose density  $>=$  MinPts
	- Core points are in the interior of a density-based cluster.



Example: If  $MinPts =$ 6 then A is a core point because its density  $= 7$  $(7>6)$ 

- Three types: core point, border point, and noise point.
	- A border point is not a core point but falls within the neighborhood of a core point.



#### Example:

- The density of B is 4 and less than  $MinPts = 6$ , so B is not a core point.
- But B falls within the neighbor of A (a core point).
- So, B is a border point.

- Three types: core point, border point, and noise point.
	- A noise point is any point that is neither a core point nor a border point.

The density of C is 3 which

is less than  $MinPts = 6$ ,

neighborhood of any core

point, so it is not a border

• So, C is a noise point..

point.

so C is not a core point.



- Steps of DBSCAN to identify clusters
	- Step 1: Label each point as either core, border, or noise point.
	- Step 2: Mark each group of Eps connected core points as a separate cluster.
	- Step 3: Assign each border point to one of the clusters of its associate core points.

### DBScan Example

Original Points



$$
Eps = 10, MinPts = 4
$$
\n
$$
10, minFs = 4
$$
\n<math display="block</math>

Mark core, border and noise points Mark connected core points



# DBScan Properties

- DBSCAN:
	- Resistant to noise and outliers
	- Can handle clusters of different shapes and sizes
	- Computational complexity is similar to K-means
- When DBSCAN does not work well
	- Varying densities
		- Can be overcome by using sampling
	- Sparse and high-dimensional data
		- Can be overcome by using topology preserving dimension reduction techniques.

- Self organizing maps are a type of Neural Network (NN).
- Unsupervised algorithm.
- Project high dimensional data onto a ndimensional display space (the feature map). – Commonly n=2
- Topology preserving mappings & clustering.
	- Data that is "similar" within the input space remain "close" to each other in the display space

- Self-organizing maps have two layers:
	- An input layer and
	- An output layer called the feature map.
- The feature map consists of neurons.
	- organized on a regular grid.
	- $-$  Unlike other ANN types, the neuro in a SOM don't have an activation function.
- Each neuron in a SOM is assigned a weight vector with the same dimensionality as the input space.



- The weights in a SOM are trained in a twostep algorithm:
	- Step 1: Competitive step
		- Every neuron is examined to calculate which one's weights is most similar to the input vector. The winning neuron is known as the Best Matching Unit (BMU).
	- Step 2: Cooperative step
		- The weights of the BMU and the weights of the neighboring neurons is updated.

- Training Algorithm:
	- 1. Each neuron's weights is initialized with random values.
	- 2. A sample is chosen at random from the set of training data.
	- 3. Find the BMU.
	- 4. Identify the neighbourhood of the BMU. The size of the neighborhood decreases over time.
	- 5. Update the weights of the BMU and all of its neighbors so that they become more similar to the sample vector. The closer a node is to the BMU, the more its weights get altered and the farther away the neighbor is from the BMU, the less it is updated.
- Step 2 through to 5 are repeated N times.
	- Normally N is a multiple of the number of training samples.

#Step 1: Prepare the data import pandas as pd import numpyas np from sklearn.utilsimport shuffle

```
df = pd.read.csv("A1 BC SEER data.csv", )df = shuffle(df)df = df[:int(df.shape[0]*0.2)] #use 20% subset for
                                 #demonstration purposes
target = df['Survival months'] #Extract the target column
#Binarize target
target = np.where(df['Survival months'] < 60, 0, \text{target})
target = np.where(df['Survival months'] >= 60, 1, target)
```
#Step 2: Preprocessthe data from sklearn.model selectionimport train test split

myseed=7 #Seed for the random number generator

```
#Remove irrelevant features, and targets from df
dropList= ['Patient ID', 'Survival months']
for item in dropList:
df.drop(item, axis=1, inplace=True)
```

```
#Scale the data?
#from sklearnimport preprocessing
#scaling = preprocessing.MinMaxScaler()
#data = scaling.fit transform(data)
```

```
#Create a train, test, and validation set
X, X tst, Y, Y tst= train test split(df, target, test size=.333,
random state=myseed)
X_trn, X_val, Y_trn, Y_val= train test split(X, Y, test size=.5,
random state=myseed)
```

```
X_trn= X_trn.to_numpy()
X_tst= X_tst.to_numpy()
X val= X val.to numpy()
```
#Step 3: Train the SOM from myminisom import MiniSom #see Moodle site for myminisom

```
#Create the SOM
som shape = (100, 100) #define the size of the som
som = MiniSom(som shape[0], som shape[1], X_trn.shape[1],
sigma=som shape[0]/2, learning rate=.9,
neighborhood function='gaussian', random seed=myseed)
#initialize the SOM, then train it
epochs=40
```

```
som.pca weights init(X trn)
som.train random(X trn, epochs * len(X trn), verbose=True)
```

```
#Find the BMU for each sample
BMU trn = np.array([som.winner(x) for x in X trn])
BMU class0 = BMU trn[Y trn==0]
BMU class1 = BMU trn[Y trn==1]
```
#Step 4: Plot some results (density map of all samples) import matplotlib.pyplot as plt from copy import copy

```
densitymap = np.zeros(som_shape)
for row in range(0,BMU trn.shape[0]):
      x, y = BMU trn[row]
      densitymap[y, x] += 1
```

```
densitymap[densitymap==0]=np.nan #mark zero values with nan
my cmap = copy(plt.cm.jet)
my cmap.set bad(color=(1,1,1)) #plot nan in white color
plt.imshow(densitymap, cmap=my_cmap, interpolation="none", 
origin="lower", aspect=0.75)
plt.colorbar()
plt.title('Mapping density')
plt.show()
```

```
#density map of all samples from class 1
import matplotlib.pyplot as plt
densitymap = np.zeros(som_shape)
for row in range(0,BMU_class1.shape[0]): 
  x, y = BMU class1[row]densitymap[y, x] += 1
```
densitymap[densitymap==0]=np.nan #mask zero values

plt.imshow(densitymap, cmap=my\_cmap, interpolation="none", origin="lower", aspect=0.75)

```
plt.colorbar()
plt.title('Mapping density (class 1)')
plt.show()
```
#### Self-Organizing Maps – An Example



### Self-Organizing Maps – An Example



### Self-Organizing Maps – An Example



# Self-Organizing Maps - Note

- Each neuron clusters samples that are mapped to it.
	- $n x m$  clusters (size of the SOM)
- A group of neurons form larger cluster
	- Cluster analysis needed to detect these.

#Compute the quantization error

qerr = som.quantization error(X trn)

qerr

7.454546962215053

#inspect some weights

#### som.get weights()[1,1]

array([1.45920861e+00, 2.94414892e+00, 1.58042285e+00, 1.98123243e+00, 5.60639639e+01, 1.49714974e+02, 2.37928820e-01, 2.06261914e+02, 5.05750682e+02, 1.57189567e+00, 1.03022154e+00, 8.98461177e+00, 1.20844111e+00, 1.33963081e+00, 5.28842982e+01, 4.74582306e+01])

- SOMs are an excellent choice for data visualization
- Many visualization techniques
	- From exploratory data analytics
	- Dimension reduction techniques
		- i.e. PCA, t-SNE, SOM,…
- Why use Self-Organizing Maps (SOMs) in BDA?
	- Topology preservation (unlike PCA)
	- Able to deal with new data & missing values (unlike t-SNE)
	- Can reduces the amount of information that needs to be evaluated
	- Produces prototypes that represent the full set of attributes with their original meaning (unlike PCA)

– …

- When not to use SOMs in BDA:
	- –When the data is very sparse
	- –When cardinality (limited resolution) of the map is a problem.
	- –When multi-core compute infrastructure is unavailable.

# Hierarchical Clustering

- Hierarchical Clustering (hclust())
	- Hierarchical agglomerative clustering
	- Hierarchical divisive clustering



- 1. Each object is initially treated as a cluster
- 2. The clusters are then combined with the most similar cluster in each step
- 3. This process is repeated until one cluster (containing all objects) exists

Computationally very expensive  $O(n^2)$  to  $O(n^3)$ and thus rarely used in Big Data analytics.





**Images Courtesy of Google Image**